higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T880(E)(N25)T

NOVEMBER EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N4

(16030164)

25 November 2016 (X-Paper)
09:00–12:00

This question paper consists of 5 pages and 1 formula sheet.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N4
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show ALL intermediate steps and simplify where possible.
5. ALL final answers must be rounded off to THREE decimal places (unless indicated otherwise)
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Use only BLUE or BLACK ink.
8. Write neatly and legibly.

QUESTION 1

1.1 1.1.1 Sketch the graph of $xy + 5 = 3$. (3)

1.1.2 Is the graph of $xy + 5 = 3$ in QUESTION 1.1.1 a function? (1)

1.1.3 Is the graph of $xy + 5 = 3$ in QUESTION 1.1.1 discontinuous or continuous? (1)

1.2 Sketch the graph of $y = \frac{\sin 2x}{2}; -180^\circ \le x \le 180^\circ$. (3)

1.3 Sketch the graph of $x^2 - y^2 = 15$. (2)

1.4 Given:

$$x + \frac{y}{2} + z = \frac{1}{2}$$
$$\frac{x}{3} - \frac{y}{4} - \frac{z}{2} = 1$$
$$\frac{x}{2} + \frac{y}{5} - \frac{z}{2} = \frac{1}{3}$$

Solve for y by using only Cramer's rule. (10)
[20]

QUESTION 2

2.1 Solve for x if $3x^2 - 18x + 39 = 0$ (4)

2.2 Given: $z = \frac{3}{10} - j\frac{30}{6}$

2.2.1 Find $\bar{z}$. (1)

2.2.2 Calculate y and θ of z. θ may only be positive. Show ALL steps. (2)

2.2.3 Indicate z and all values calculated in QUESTION 2.2.2 on an Argand diagram. (3)

2.3 Fully factorise the following:

$$(2x + 5y)^3 - (x - 3y)^3$$ (3)

2.4 Make a the subject of the formula if $P + \cos x = te^{at} \sin x$ (4)

2.5 Solve for y if $5^{2y+2} = 3^{5y-1}$ (3)
[20]

QUESTION 3

3.1 If x and y are angles in the second quadrant and $\tan x = -\frac{1}{2}$ and $\tan y = -\frac{1}{3}$ determine the value of $\tan(x+y)$ without the use of a calculator. (3)

3.2 Prove that $\frac{\sin 2\theta}{2} + \frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = 1$ (5)

3.3 Given:

$$\cos\frac{1}{2}x = \pm\sqrt{\frac{\cos x + 1}{2}}$$

Determine without the use of a calculator the value of $\cos 22{,}5^\circ$ (4)

3.4 Simplify:

$$\frac{1+2\sin\theta}{\cos\theta + \sin 2\theta}$$ (3)

3.5 Solve for θ if $5\tan^2\theta + 8\sec\theta = 1$; $0^0 \le \theta \le 360^0$ (5)

[20]

QUESTION 4

4.1 Differentiate the following in terms of x:

$$y = 3(\pi^x) + \frac{1}{3}\log_3 x - \frac{1}{3\tan x} - \sqrt{x} + \frac{3}{\sin x}$$ (5)

4.2 Given: $y = \sqrt{x}(x^2 + x + 1)$

Differentiate by use of the product rule. (4)

4.3 Given: $y = 3x^3 + 9x^2 + 3$

Solve for x if $\frac{d^2y}{dx^2} = 0$. (4)

4.4 Given: $y = 2x^3 - 8x$

Determine, by the use of differentiation, the co-ordinates of the maximum and the minimum turning points of the given function. (7)

[20]

QUESTION 5

5.1 5.1.1 Sketch and indicate the area included by the graph of $y = \sqrt{x}$ and $x = 3$ in the first quadrant. Also, indicate the representative strip used to calculate the area shown. (3)

5.1.2 Calculate, using integration, the value of the area indicated in QUESTION 5.1.1 above. (4)

5.2 Evaluate the following:

$$\int_0^2 \left(2 - \sqrt{x}\right) dx$$ (3)

5.3 Integrate the following:

$$\int \left[(3x)^2 - \frac{3}{\sqrt{x^3}} + 3.7^{-3x} - 4\tan\frac{4}{x} - \frac{3}{\cot x \cos x} - 1 \right] dx$$ (7)

5.4 Simplify $\int (\cos 2y + \sin 2y) dy$ (3)

[20]

TOTAL: 100

MATHEMATICS N4

FORMULA SHEET

$a^x = b \Leftrightarrow \log a^x = \log b$

$\ln x = \log_e x$

$\left(r\left|\underline{\theta}\right.\right)^n = r^n\left|\underline{n\theta}\right.$ $\quad a+bj = c+dj \Leftrightarrow a = c and b = d$

$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$

$\cos(a \pm b) = \cos a \cos \mp \sin a \sin b$

$\tan(a \pm b) = \dfrac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \cos ec^2 x$

$1 + \tan^2 x = \sec^2 x$

y	$\dfrac{dy}{dx}$
an^n	nax^{n-1}
ka^x	$ka^x \ln a$
$k \ln x$	$\dfrac{k}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\cos ec^2 x$
$\sec x$	$\sec x \tan x$
$\cos ecx$	$-\cos ecx \cot x$

$y = u(x) \cdot v(x)$

$\Rightarrow \dfrac{dy}{dx} = u(x)v^1(x) + u^1(x)v(x)$

$y = \dfrac{u(x)}{v(x)}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{v(x)u^1(x) - u(x)v^1(x)}{[v(x)]^2}$

$\dfrac{dy}{dx} = \dfrac{dy}{du} x \dfrac{du}{dx}$

$\int ax^n dx = \dfrac{ax^{n+1}}{n+1} + C$

$\int \dfrac{a}{x} dx = a\ell nx + c$

$\int ka^x dx = \dfrac{ka^x}{\ell na} + c$

$A_{ox} = \int_a^b y dx$

$\int \sin x dx = -\cos x + c$

$\int \cos x dx = \sin x + c$

$\int \tan x dx = \ell n \sec x + c$

$\int \sec x dx = \ell n(\sec x + \tan x) + c$