higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T890**(E)**(N22)T
**NOVEMBER EXAMINATION**

**NATIONAL CERTIFICATE**

# MATHEMATICS N5

(16030175)

**22 November 2016 (X-Paper)**
**09:00–12:00**

**Scientific calculators may be used.**

**This question paper consists of 6 pages and a formula sheet of 5 pages.**

# DEPARTMENT OF HIGHER EDUCATION AND TRAINING
# REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N5
TIME: 3 HOURS
MARKS: 100

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**INSTRUCTIONS AND INFORMATION**

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show ALL intermediate steps and simplify where possible.
5. ALL final answers must be rounded off to THREE decimal places
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Questions must be answered in blue or black ink.
8. Write neatly and legibly.

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## QUESTION 1

1.1 Determine the following limits:

1.1.1 $\lim\limits_{x\to\infty} \dfrac{x}{\ln x} - \dfrac{1}{\ln x}$ (3)

1.1.2 $\lim\limits_{x\to 0} \dfrac{x^3 - 2x - 5}{x^2 - 2x}$ (2)

1.2 Determine whether $f(x) = \sqrt{4x - \dfrac{1}{2}}$ is continuous at $x = -1$. (2)

**[7]**

## QUESTION 2

2.1 Determine the derivative of $f(x) = \sqrt[3]{2x}$ from first principles. (5)

2.2 Determine $\dfrac{dy}{dx}$ if:

2.2.1 $y = \ln^2 \dfrac{3x}{e^x} - \ln 3x^2$ (4)

2.2.2 $y = 5\arccos ec(x \sin 2x)$ (3)

2.2.3 $y = 4\sqrt{\cos^3(x^2 - x - 1)}$ (3)

2.3 Given: A particle moves along a line such that displacement (in meters) in time (t seconds) is given by

$s(t) = t^{\cos t}$

Determine the velocity at $t = \dfrac{\pi}{4}$ seconds with the aid of logarithmic differentiation. (5)

2.4 Given: $\cos(x + y) + \sin x + \sin y = 2$

Make use of implicit differentiation to calculate $\dfrac{dy}{dx}$. (4)

**[24]**

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**QUESTION 3**

3.1 Given: $f(x) = 3x^3 - 4x^2 - 2x + 2$

3.1.1 Determine the coordinates of the turning points of $f(x)$. (3)

3.1.2 Verify by using a table that the equation $0 = 3x^3 - 4x^2 - 2x + 2$ has a root between the points $x = 1$ and $x = 2$.

Use values on the table: $-1 \leq x \leq 2$ (2)

3.1.3 Hence, make a neat sketch of the graph of the function $f(x)$. (3)

3.1.4 If the positive root of $f(x)$ is estimated as 1,7, use Taylor's/Newton's method to determine a better approximation of this root. (2)

3.2 A farmer wants to enclose a field with length and breadth $x$ meter and $y$ meter respectively.

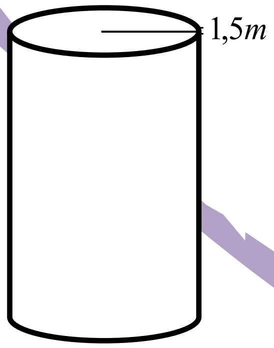
The cost of fencing is R36,00/m for the length and R45,00/m for the breadth. He has an amount of R56 000,00 available.

3.2.1 Give the formula of the area and the cost in terms of $x$ and $y$. (2)

3.2.2 Calculate the dimensions of the field with maximum area. (5)

3.3 A fluid flows into a cylindrical tank of radius 1,5 m at a rate of 3 $m^3$/s.

Calculate how fast the surface is rising.




HINT: $V = \pi . r^2 . h$ (4)

**[21]**

## QUESTION 4

4.1 Determine $\int \frac{1-\cos 2x}{-2x+\sin 2x}$ (2)

4.2 Determine $\int y dx$ in each of the following cases:

4.2.1 $\int 2x.e^{2x} dx$ (3)

4.2.2 $\int \frac{2x^3-4}{x+1} dx$ (5)

4.2.3 $\int \frac{\sin^3 3x}{\sin 3x} dx$ (3)

4.2.4 $\int 5x^2\sqrt{2x^3+1}\, dx$ (3)

4.3 Determine $\int y dx$ by resolving the integrand into partial fractions:

$$\int \frac{4x^2-3x+1}{4x^2-1}$$ (5)

4.4 Determine: $\int \sin(\arcsin x^{-1}) dx$ (2)

**[23]**

## QUESTION 5

5.1 Given: The curves $x = -y^2 + 4$ and $x = 4 - 2y$

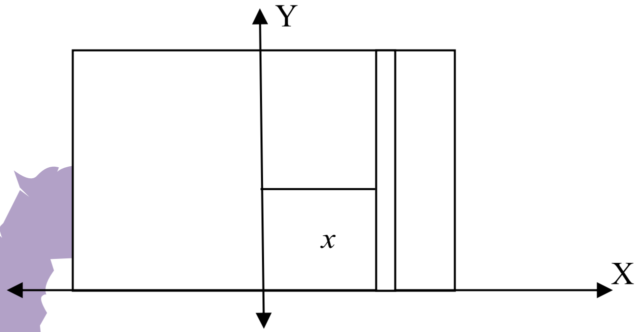
5.1.1 Sketch the TWO graphs. Shade the area enclosed by the two graphs and show the representative strip with dimensions $x$ and $dy$. (3)

5.1.2 Calculate the magnitude of the enclosed area towards the Y-axis. (4)

5.1.3 Calculate the volume generated when this area rotates about the Y-axis. (4)

5.2 Determine the second moment of mass of a rectangular lamina of mass (m) about the axis parallel to one side of the lamina, which bisects the lamina.

The distance from the representative strip to the axis is $x$. The lamina's length and breadth is $a$ and $b$ respectively.




(4)
**[15]**

**QUESTION 6**

6.1 Solve the differential equation $e^{2x+3y}.\frac{dx}{dy}=e^{-5x+8y}$ (4)

6.2 Determine the general solution of the differential equation:

$$\frac{d^2y}{dx^2}=2.10^{4x}+\frac{1}{\sin^2 2x}-4$$

(6)
**[10]**

**TOTAL: 100**

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**MATHEMATICS N5**

**FORMULA SHEET**

Any applicable formula may also be used.

**TRIGONOMETRY**

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = \frac{1}{2} [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = \frac{1}{2} [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = \frac{1}{2} [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = \frac{1}{2} [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$ ; $\sin x = \dfrac{1}{\text{cosec } x}$ ; $\cos x = \dfrac{1}{\sec x}$

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**BINOMIAL THEOREM**

$$(x+h)^n = x^n + nx^{n-1}\,h + \frac{n(n-1)}{2!}x^{n-2}\,h^2 + \dots$$

**DIFFERENTIATION**

$$e = -\frac{f(a)}{f^{'}(a)}$$

$$r = a + e$$

**PRODUCT RULE**

$$y = u(x)\,.\,v(x)$$

$$\frac{dy}{dx} = u\,.\,\frac{dv}{dx} + v\,.\,\frac{du}{dx}$$

$$= u \cdot v^{'} + v \cdot u^{'}$$

**QUOTIENT RULE**

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v\,.\,\frac{du}{dx} - u\,.\,\frac{dv}{dx}}{v^2}$$

$$= \frac{v \cdot u' - u \cdot v'}{v^2}$$

**CHAIN RULE**

$$y = f(u(x))$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

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| $f(x)$ | $\frac{d}{dx} f(x)$ | $\int f(x)dx$ |
|---|---|---|
| $ax^n$ | $nax^{n-1}$ | $\frac{ax^{n+1}}{n+1} + c$ |
| $a$ | 0 | $ax + c$ |
| $e^x$ | $e^x$ | $e^x + c$ |
| $a^x$ | $a^x.\ln a$ | $\frac{a^x}{\ln a} + c$ |
| $\log_e x$ | $\frac{1}{x}$ | — |
| $\log_a x$ | $\frac{1}{x \ln a}$ | — |
| $\sin x$ | $\cos x$ | $-\cos x + c$ |
| $\cos x$ | $-\sin x$ | $\sin x + c$ |
| $\tan x$ | $\sec^2 x$ | $\ln(\sec x) + c$ |
| $\cot x$ | $-\mathrm{cosec}^2 x$ | $\ln(\sin x) + c$ |
| $\sec x$ | $\sec x \cdot \tan x$ | $\ln[\sec x + \tan x] + c$ |
| $\mathrm{cosec}\, x$ | $-\mathrm{cosec}\, x \cdot \cot x$ | $\ln(\mathrm{cosec}\, x - \cot x) + c$ |
| $\sin^{-1} x$ | $\frac{1}{\sqrt{1-x^2}}$ | — |
| $\cos^{-1} x$ | $\frac{-1}{\sqrt{1-x^2}}$ | — |
| $\tan^{-1} x$ | $\frac{1}{1+x^2}$ | — |
| $\cot^{-1} x$ | $\frac{-1}{1+x^2}$ | — |
| $\sec^{-1} x$ | $\frac{1}{x\sqrt{x^2-1}}$ | — |
| $\mathrm{cosec}^{-1} x$ | $\frac{-1}{x\sqrt{x^2-1}}$ | — |

| $f(x)$ | $\frac{d}{dx} f(x)$ | $\int f(x)dx$ |
|---|---|---|
| $\frac{1}{\sqrt{a^2 - x^2}}$ | — | $\sin^{-1}\left(\frac{x}{a}\right) + c$ |
| $\frac{1}{a^2 + x^2}$ | — | $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$ |
| $\frac{1}{x\sqrt{x^2 - a^2}}$ | — | $\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$ |
| $\sqrt{a^2 - x^2}$ | — | $\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2 - x^2} + c$ |
| $\frac{1}{x^2 - a^2}$ | — | $\frac{1}{2a} ln\left(\frac{x-a}{x+a}\right) + c$ |
| $\frac{1}{a^2 - x^2}$ | — | $\frac{1}{2a} ln\left(\frac{a+x}{a-x}\right) + c$ |

**INTEGRATION**

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$$

$$\int [f(x)]^n f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

$$\frac{f(x)}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$$

$$\frac{f(x)}{(x+a)^n} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)^3} + \cdots \frac{Z}{(x+a)^n}$$

**APPLICATIONS OF INTEGRATION**

**AREAS**

$$A_x = \int_a^b y dx \; ; \; A_x = \int_a^b (y_1 - y_2) dx$$

$$A_y = \int_a^b x dy \; ; \; A_y = \int_a^b (x_1 - x_2) dy$$

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**VOLUMES**

$$V_x = \pi \int_a^b y^2 dx \;;\; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx$$

$$V_y = \pi \int_a^b x^2 dy \;;\; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy$$

**SECOND MOMENT OF AREA**

$$I_x = \int_a^b r^2 dA \;;\; I_y = \int_a^b r^2 dA$$

**MOMENTS OF INERTIA**

Mass = density × volume

$M = \rho V$

DEFINITION: $I = m\, r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

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