higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(First Paper)
NQF LEVEL 3

(10501053)

4 November 2019 (X-Paper)
09:00–12:00

This question paper consists of 6 pages, 3 diagram sheets and 1 formula sheet.

Copyright reserved

TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Write neatly and legibly.

Copyright reserved

QUESTION 1

1.1 Write each of the following imaginary numbers in their simplest form:

1.1.1 $i^{\sqrt{64}}$

1.1.2 $(2i)^5 \times i^9$

(2 × 2) (4)

1.2 Simplify each of the following without using a calculator:

1.2.1 $(12{,}6 + j4{,}9) + (-1{,}3 - j9{,}6)$ (2)

1.2.2 $(6 - i7)(6 + i7)$ (2)

1.2.3 $\dfrac{-7 - i2}{-4 + i}$ (3)

1.2.4 $3(\cos 25° + i \sin 25°).8(\cos 200° + i \sin 200°)$ (2)

1.2.5 $\dfrac{4(\cos 190° + i \sin 190°)}{2(\cos 70° + i \sin 70°)}$ (2)

1.3 Convert each of the following complex numbers into polar form and express the argument θ as a positive angle:

1.3.1 $z = -6 + i9$ (3)

1.3.2 $z = \dfrac{-7 - i8}{2 - 3i}$ (5)

1.4 Simplify each of the following and express the answer in the form $a + ib$. Show ALL calculations.

1.4.1 $\dfrac{12\underline{|45°}.6\underline{|180°}}{8\underline{|20°}}$ (4)

1.4.2 $\dfrac{3{,}8(\cos 120° + i \sin 120°)}{2{,}5 \cos 45°}$ (3)

[30]

QUESTION 2

2.1 Solve for x in the following inequality:

$x^2 - 8x + 23 \leq 8$ (5)

2.2 Solve for x by completing the square:

$5x^2 - 3 = 2x$ (4)

2.3 Solve for x and y in:

$y + 2x = 2$
$y^2 + 2x^2 = 3xy$ (4)

2.4 Simplify each of the following:

2.4.1 $\dfrac{a^2 - a}{a - 2} \cdot \dfrac{a + 1}{a^2 + 4a} \div \dfrac{a^2 - 3a - 4}{a^2 - 16}$

2.4.2 $\dfrac{y}{y^2 - 16} - \dfrac{y + 1}{y^2 - 5y + 4}$

(2 × 4) (8)

2.5 The following constraints are given for a linear programming problem:

$x \leq 6$
$y \leq 7$
$5x + 3y \leq 36$
$x - 6y + 6 \leq 0$

2.5.1 Use DIAGRAM SHEET 1 (attached) and sketch the graph with the given constraints. (4)

2.5.2 Determine the feasible region by shading. (1)

2.5.3 Find the values of x and y that will maximize the profit if the objective function is $P = 100\,x + 200\,y$ by means of a boundary search. (4)

[30]

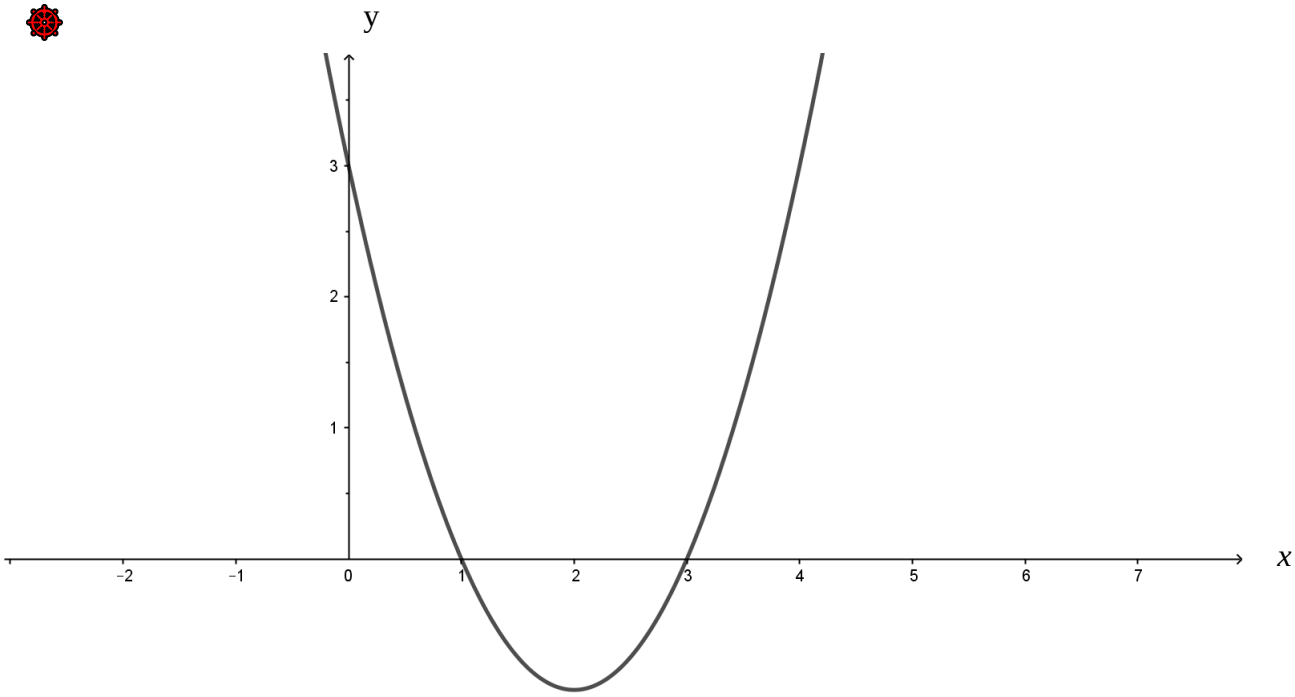
QUESTION 3

3.1 Given $f(x): 2y - x = -1$ and $g(x) = \frac{4}{x-4} + 2$

Use DIAGRAM SHEET 2 and sketch the graphs of $f(x)$ and $g(x)$ on the same system of axes. Indicate the asymptotes of $g(x)$ (8)

3.2 In the figure below the graph of $y = ax^2 + bx + q$ is shown.

Study the graph and answer the questions.



3.2.1 Find the equation for the graph $y = ax^2 + bx + q$. (4)

3.2.2 Calculate the turning point of the graph $y = ax^2 + bx + q$. (4)

3.3 Use DIAGRAM SHEET 3 (attached) and sketch the graph of $y = \left(\frac{1}{2}\right)^x$ where $-3 \leq x \leq 3$ (4)

[20]

QUESTION 4

4.1 If $s(t) = t^2 + 3t$, find the derivative $\frac{ds}{dt}$ from first principles. (4)

4.2 Determine the following limit:

$$lim_{x\to\infty}\frac{4-x^2}{x^3-2}$$

(3)

4.3 Use differentiation rules to determine $\frac{dy}{dx}$ of each of the following. Leave the answer with a positive exponent and in surd form where applicable.

4.3.1 $y = \frac{1}{4}x^8 - \frac{1}{2}x^4 + 2$

4.3.2 $y = \frac{2}{x} - \sqrt{x}$

4.3.3 $y = (x+2)(3x-5)$

(3 × 3) (9)

4.4 Find the greatest product that can be obtained by multiplying two numbers of which the sum is 32.

HINT: Make the one number (x) and the other number $(32 - x)$. (4)

[20]

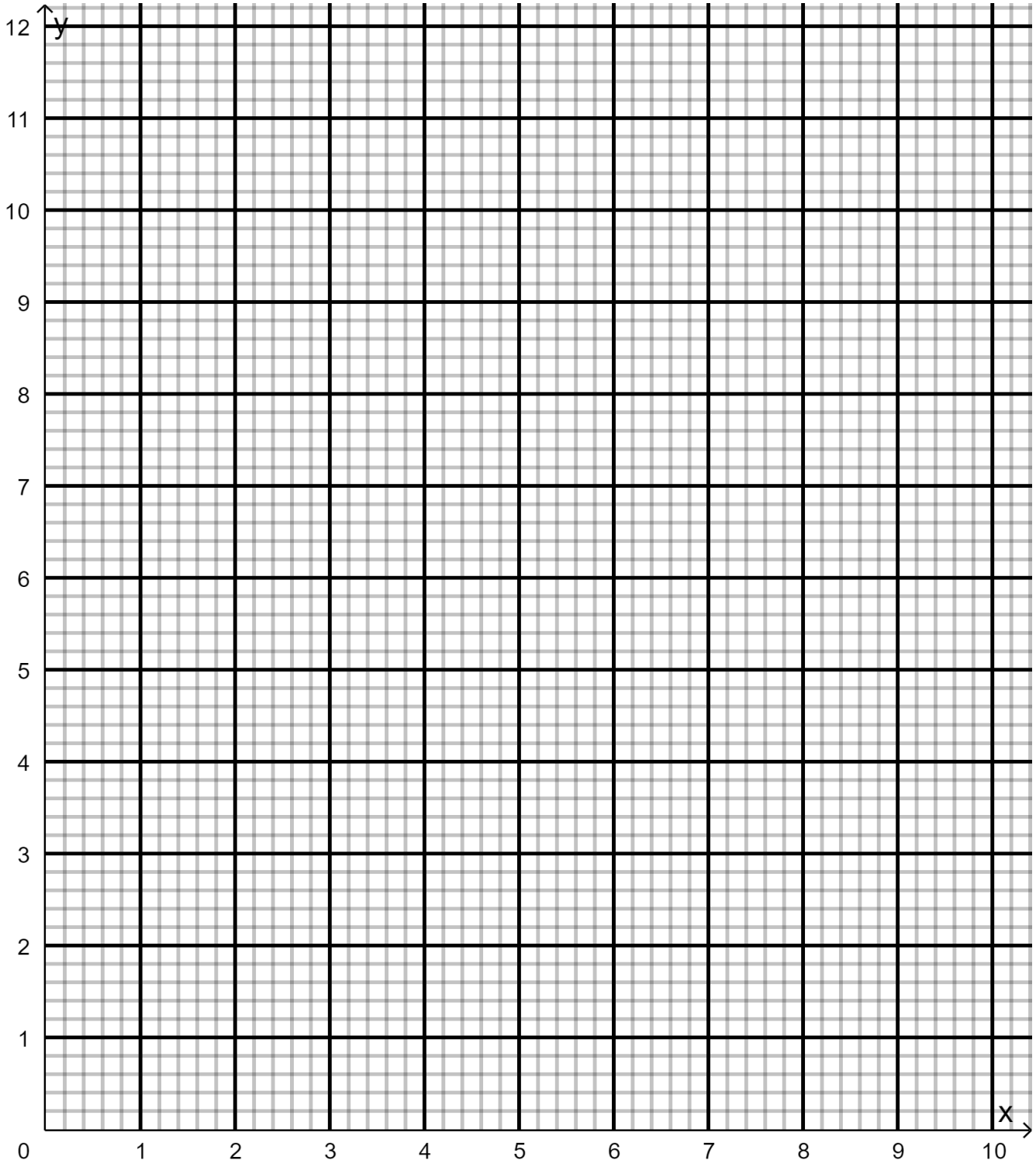
TOTAL: 100

Copyright reserved

DIAGRAM SHEET 1

EXAMINATION NUMBER:

QUESTION 2.5.1



Copyright reserved

DIAGRAM SHEET 2

EXAMINATION NUMBER: | | | | | | | | | | | | |

QUESTION 3.1

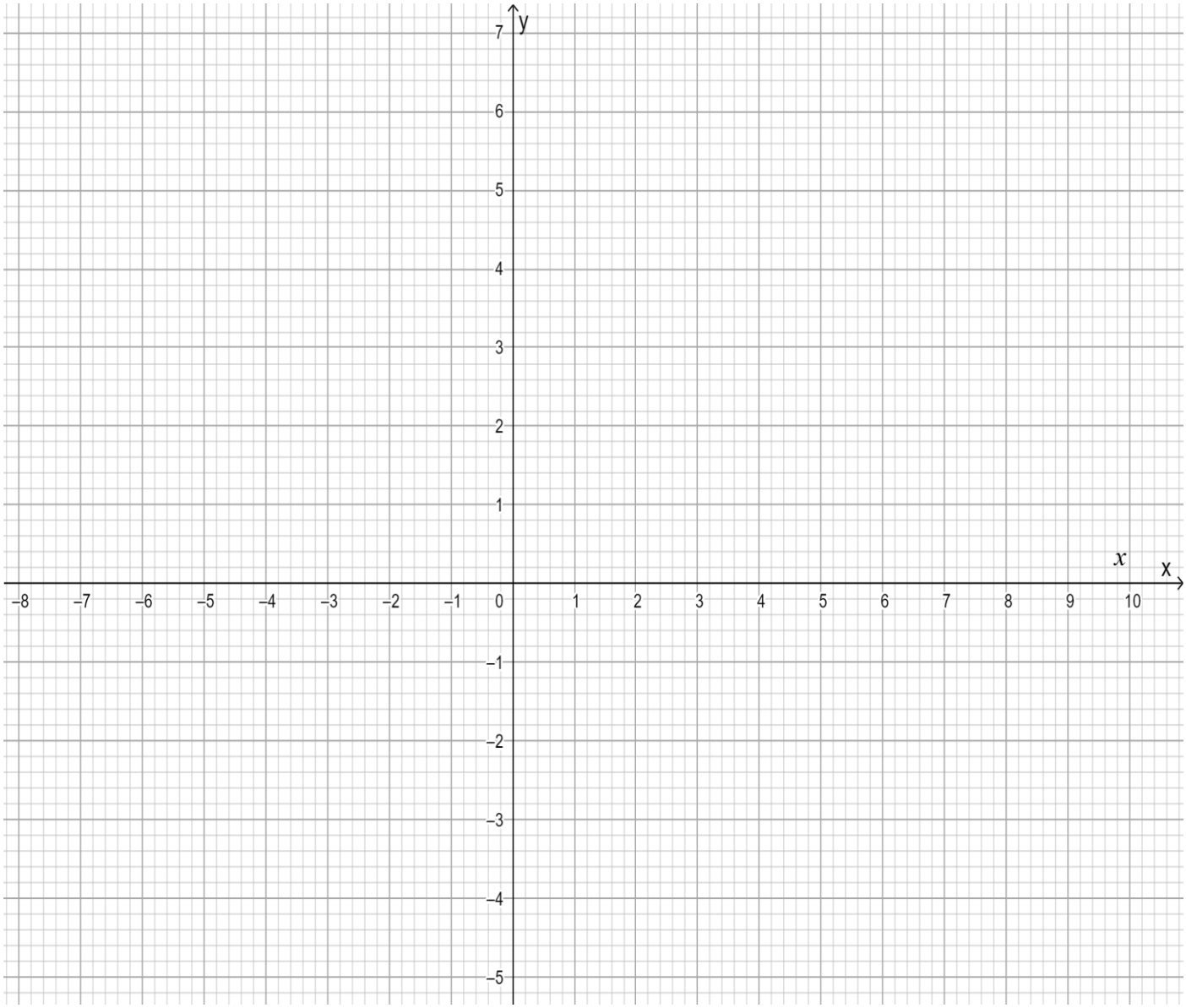
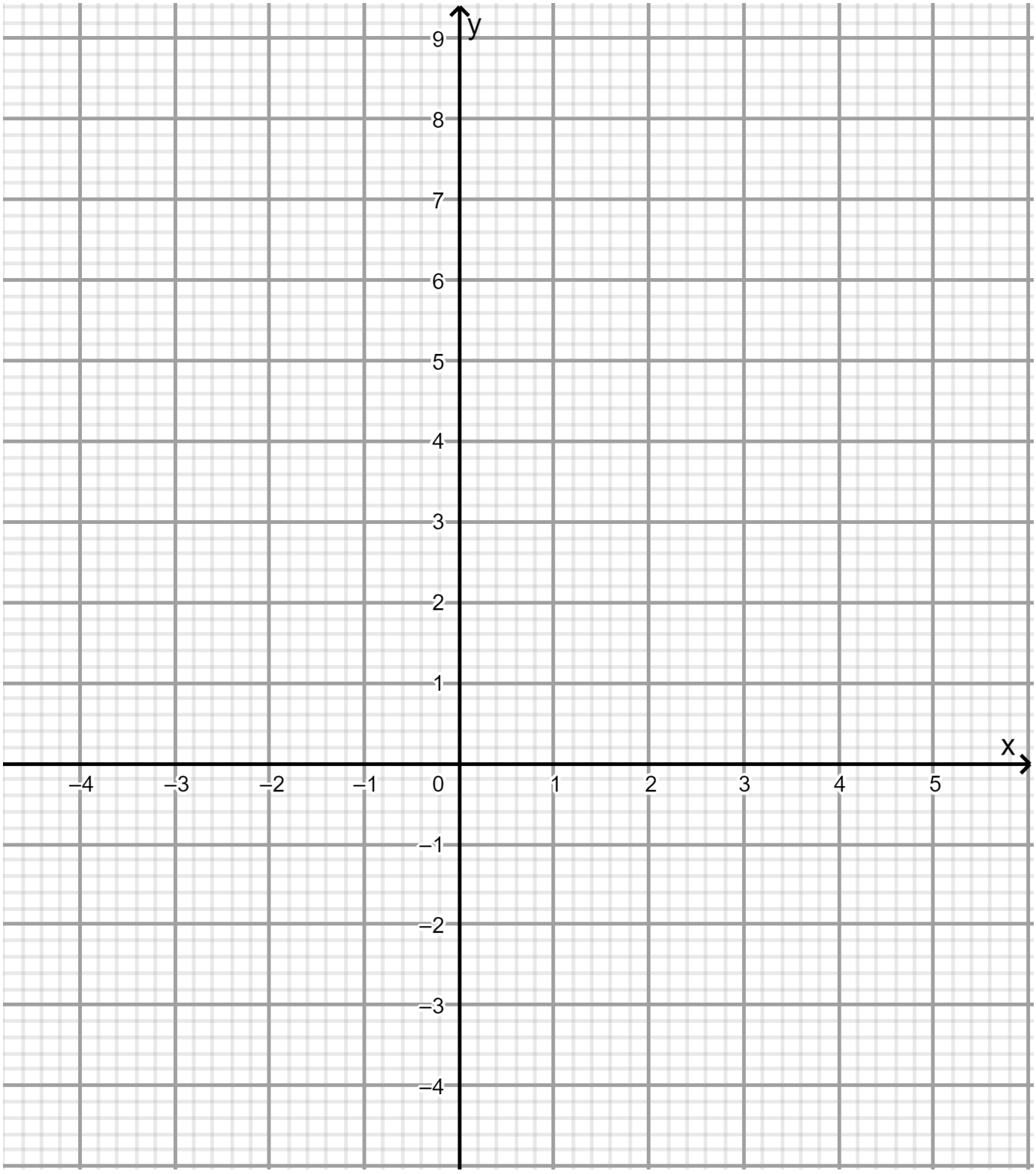


DIAGRAM SHEET 3

EXAMINATION NUMBER: | | | | | | | | | | | | |

QUESTION 3.3



Copyright reserved

FORMULA SHEET

1. $z = r\cos\theta + r\,j\sin\theta$

2. $z = a \pm bj$ or $z = a \pm bi$ where $i = j = \sqrt{-1}$

3. $r = \sqrt{a^2 + b^2}$ or $r = \sqrt{z \times \bar{z}}$

4. $\alpha = \tan^{-1}\left(\dfrac{b}{a}\right)$

5. $r \angle\theta = r\cos\theta$

6. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

7. $y = ax^2 + bx + c$

8. $y = a(x - p)^2 + q$

9. $y = a(x - x_1)(x - x_2)$

10. $y = \dfrac{a}{(x + p)} + q$

11. $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

12. $\dfrac{d}{dx} x^n = nx^{n-1}$

13. $\dfrac{d}{dx} k = 0$

14. $Dx[kf(x)] = kDx[f(x)]$

15. $Dx[f(x) \pm g(x)] = Dx[f(x)] \pm Dx[g(x)]$

Copyright reserved